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DISCUSSION OF PLASTICITY OF METALS—MATHEMATICAL THEORY AND STRUCTURAL APPLICATIONS

(*Published in August, 1950*)

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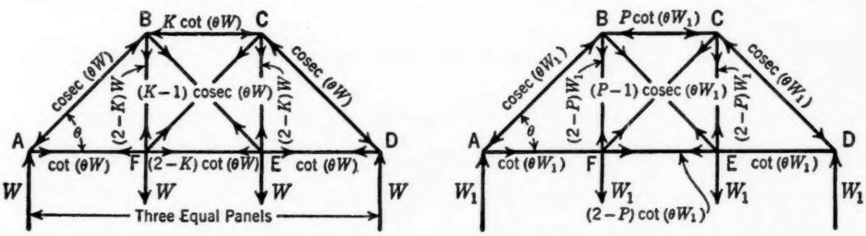
DISCUSSION

R. W. STEED[15].—For some time the writer has been engaged in an investigation into the design of those triangulated steel structures which rely, primarily, for their resistance to loading on longitudinal forces. Each member is subjected to simple tension or compression. Ideal plasticity has been assumed as suggested in the paper.

It is not sufficient to choose an arbitrary load distribution for multiredundant structures under ultimate loading, unless the mechanism of failure is known, or unless recourse is made to the solution of statically indeterminate elastic equations. There is the possibility of compression members buckling before the ultimate load is reached. Insufficient experimental knowledge is available concerning the behavior of buckled compression members, as further load is added to a structure, for this premature buckling to be permissible. In the case of some structures there is also the possibility of local unloading after loading, even while all loads applied to a structure increase in ratio.

Triangulated structures, designed on the ultimate theory, having one or two symmetrically loaded, internal or external redundancies, can be analyzed as follows: Referring to Fig. 11: let W_u be the ultimate design load; let W_w be the



(a) K VARIES BETWEEN 1 AND 2 (b) P VARIES BETWEEN 1 AND 2

FIG. 11

working load, such that $W_u = W_w\lambda$, λ being a selected load factor; and let θ be the slope angle of diagonal members. Finally, let the ultimate load in BC be $K\ W_u \cot\theta$ in which K is some number, in this example varying between 1 and 2. The ultimate loads in the remaining members are as shown in Fig. 11(a).

The areas of members BF, CE, EF, BE, and CF are determined by dividing the ultimate loads in the members by σ_y, the yield stress of the material. As member BC is in compression, the buckling stress will be less than σ_y and can be denoted by σ_y/N, in which N is some number. The area of member BC, then, will be $K\ W_u\ N \cot\theta/\sigma_y$.

As both of the applied loads are increased incrementally, all members are in an elastic state until at some applied load W_y one member yields or buckles. Let the load in member BC now be $P\ W_y \cot\theta$, and the loads in the other members become as shown in Fig. 11(b). If the symbol Q be made to represent

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the ratio $\dfrac{\text{load capacity of a member}}{\text{elastic distribution for the member}}$, the following relations may be written:

$$Q_{BC} = \frac{W_u K}{P} \qquad (17a)$$

$$Q_{BF} = Q_{CE} = Q_{EF} = W_u \frac{2 - K}{2 - P} \qquad (17b)$$

and

$$Q_{CF} = Q_{BE} = W_u \frac{K - 1}{P - 1} \qquad (17c)$$

If P is greater than K, Eqs. 17*a* and 17*c* are less than W_u, and Eq. 17*b* is greater than W_u. Members BE and CF will then be the first to yield, as $\dfrac{K - 1}{P - 1} < \dfrac{K}{P}$. If P is less than K, Eqs. 17*a* and 17*c* are greater than W_u, and Eq. 17*b* is less than W_u. Under the latter conditions members BF, CE, and EF will be the first to yield.

From a strain energy analysis of the structure it can be shown that, assuming no lack of fit of the members:

$$-2\frac{2 - P}{2 - K} - \cot^2\theta \frac{2 - P}{2 - K} + 2\operatorname{cosec}^2\theta \frac{P - 1}{K - 1} + \frac{\cot^2\theta\, P}{K\, N} = 0 \qquad (18)$$

If $P = K$ in Eq. 18, $-2 - \cot^2\theta + 2\operatorname{cosec}^2\theta + \dfrac{\cot^2\theta}{N}$ should equal 0. In other words, $-1 + \operatorname{cosec}^2\theta + \dfrac{\cot^2\theta}{N}$ should equal 0. For all values of θ between 0° and 90° (the left hand side being always positive), this is an impossibility, regardless of the value of N. If P is greater than K, the quantity—

$- < 2 - < \cot^2\theta + > 2\operatorname{cosec}^2\theta + > \dfrac{\cot^2\theta}{N}$—should equal 0.

The positive value of the left hand side is now even greater than before. If P is less than K the quantity— $- > 2 - > \cot^2\theta + < 2\operatorname{cosec}^2\theta + < \dfrac{\cot^2\theta}{N}$— should equal 0. Since this is a possibility, P is less than K.

Theoretically, members BF, CE, and EF, as a group, will always yield first for any value of θ. For practical purposes the areas of members BF and CE could be made slightly greater than their theoretical areas so that the only member to be in a plastic state as the load is increased from W_y to W_u would be member EF. With the application of the full design load, W_u, all members would be become plastic and the structure would fail. It is appreciated that the theoretical areas here used would be adjusted to practical sizes but this would tend only to increase the ultimate failing load and thus the load safety factor for the structure. Using a similar argument, it can be shown that, whatever the value of α for a structure such as that shown in Fig. 3(*b*), designed by ultimate theory, the middle member will always yield before either of the other two members.

The writer has applied this method successfully to investigate many well known types of structures with one, or two symmetrically loaded, internal or external redundancies, and designed by ultimate theory. Structures investigated include those shown in Fig. 12.

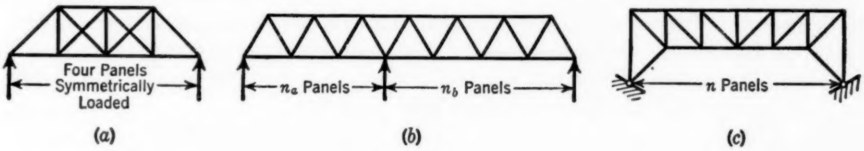

FIG. 12

It will be appreciated that, although symmetrical loading has been used in the foregoing example, the method can be used for any design loading, either symmetrical or asymmetrical, on a frame having one redundancy.

By using generalized expressions, the writer has extended the method shown in the example to investigate several well known types of structures having multiple redundancies and designed by ultimate theory.

JACOB KAROL,[16] M. ASCE.—In his comprehensive review of the theory of plasticity of metals, Mr. Drucker has made many cautious generalizations which are difficult to appreciate. Emphasizing the need for a general theory of plasticity, he disregards solutions that are possible using existing theory. In particular cases, it appears that the naive approach of the practical engineer to stress problems in general is as consonant with reality as the "more formal and precise approach" which the author states is desirable.

Two problems discussed in detail by the author will be commented upon: (1) The limit design of I-beams and (2) the structural stability of columns.

Some concern is expressed by Mr. Drucker regarding the failure of others[3,4] to consider the effect of shear stress in the limit design of I-beams. Two generalizations in the paper can be used to show that this concern may be unwarranted. First, it is stated (under the heading, "Limit Design") "* * * that any body (and therefore a structure in particular) will support the maximum load that it possibly can." This statement seems axiomatic to the writer. Furthermore, under the heading, "The Path of Loading," the author states that "* * * unloading may (and will) often take place locally."

The important fact not considered by the author is that the web of an I-beam carries practically all the shear and that the flanges carry practically all the moment. Hence, it seems reasonable to assume that in the plastic range, if necessary (and one must repeat the "if necessary"), the shear will be resisted on a part of the web at the shear yield stress, and the moment will be resisted by the remainder of the section at the tensile (and compressive) yield stress. Failure of the I-beam, fixed-ended and uniformly loaded, will be assumed to

[16] Design Engr., Kansas City, Mo.

[3] "Theory of Inelastic Bending with Reference to Limit Design," by Alexander Hrennikoff, *Transactions*, ASCE, Vol. 113, 1948, pp. 213–247.

[4] "Ueber das Verhalten statisch unbestimmter Konstruktionen aus Stahl nach Ueberschreitung der Elastizitaetsgrenze," by W. Prager, *Bauingenieur*, Vol. 14, 1933, pp. 65–67.

take place when the outer fiber stress at the center section of the beam has just reached the tensile yield stress and when the end sections have yielded in shear, moment, or in a combination of shear and moment. The compression flanges are considered to be supported against lateral buckling. Since the failure load is the load at which the center section just begins to yield, one can calculate the deflections of the beam as indicated by Alexander Hrennikoff.[3]

The load carried by a fixed-ended I-beam is given by the expression

$$W = 2\,\tau_y\,A_w\,\frac{y}{d} \qquad (19)$$

in which

$$\frac{y}{d} = \frac{4\,\sqrt{3}\,(S + 2\,Q)}{A_w\,d\,l/d}\left[1 - \frac{A_w\,d}{8\,Q}\left(\frac{y}{d}\right)^2\right] \qquad (20)$$

In addition to the symbols in the paper, in Eqs. 19 and 20 A_f = area of one flange; A_w = area of the web = $t_w\,d$; d = depth of beam; l = span length of beam; Q = static moment of a cross section; S = section modulus of cross section; t_w = web thickness; y = height of web stressed in shear; and τ_y = shear yield stress = $\sigma_y/\sqrt{3}$.

Since the ratio y/d cannot exceed unity, for low values of l/d the capacity of the beam is limited by its strength in shear. As a matter of interest, the formula for load, neglecting the effect of shear on the moment capacity, will also be given:

$$W = \frac{8\,\sigma_y}{l/d}\left(\frac{S + 2\,Q}{d}\right) \qquad (21)$$

The graph for an 18I54.7 (the same section used by Mr. Hrennikoff[3]) is shown as curves I and II in Fig. 13, abscissas representing the ratio l/d and ordinates representing W/W_{max}, in which W_{max} is the yield capacity of the beam in shear.

The beginning of plastic action is reached at the critical section in one of three ways: (1) For short spans, when the shear stress at the neutral axis reaches the yield point; (2) for intermediate spans, when the apparent stress at the junction of the web and the flange (using the distortion-energy theory of failure) reaches the tensile yield point; and (3) for long spans, when the extreme fiber stress in bending reaches the tensile yield point. The corresponding expressions for load are: Short spans—

$$W = 2\,\tau_y\,A_w\,\frac{S}{2\,Q} \qquad (22a)$$

intermediate spans—

$$W = \frac{2\,\tau_y\,A_w\,S}{A_f\,d\left[1 + \frac{1}{3}\left(\frac{A_w\,l}{6\,A_f\,d}\right)^2\right]^{\frac{1}{2}}} \qquad (22b)$$

long spans—

$$W = \frac{12\,\sigma_y\,S/d}{l/d} \qquad (22c)$$

The loads as ratios of W_{max} are also shown in Fig. 13 as curve III. Plastic failure takes place in the region between curve II and curve III. The simple theory resulting in curve I seems to be a good approximation despite its lack of formality and precision.

The author's general discussion of structural stability is commendably concise and clear. One should add the important point that, in the range of slenderness ratios used in design, the difference between the buckling loads computed by the reduced modulus theory and that computed by the tangent modulus theory is small indeed. The limits thus set by theory are all the assurance the designer needs of the validity of his design procedure. Such

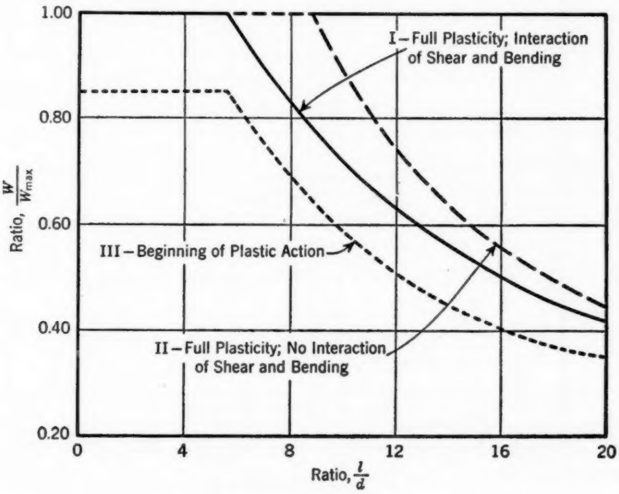

FIG. 13.—FAILURE LOADS FOR 18I54.7

procedure neglects the specific modes of failure which the author states (paragraph containing Eqs. 9*a* and 9*b*) indicate "* * * the differences between the old and new viewpoints." It will now be shown that the new viewpoint is not significant in practical design.

The apparent strain in the inclined bars in Fig. 7 at which collapse will take place may be written directly from Eqs. 9*a* and 9*b*. For no side motion of the apex:

$$\frac{F}{A\,E_t} = \cot^2 \alpha \qquad (23a)$$

For side motion of the apex:

$$\frac{F}{A\,E_t} = \tan^2 \alpha \qquad (23b)$$

The third mode of failure (the Shanley mode of buckling of the bars themselves) is given by the expression:

$$\frac{F}{A\,E_t} = \frac{\pi^2}{(L/r)^2} \qquad (23c)$$

The zone of application of each mode of buckling is shown in Fig. 14, using the slenderness ratio as abscissa and the angle α as ordinate. It is seen that the ordinary tangent modulus theory is critical for determining the buckling load for any practical combination of slenderness ratio and frame angle, and is thus the most important, although the author's parenthetical mention of it might indicate otherwise.

This discussion is not intended to depreciate any efforts, experimental or theoretical, which may be made toward the development of a comprehensive

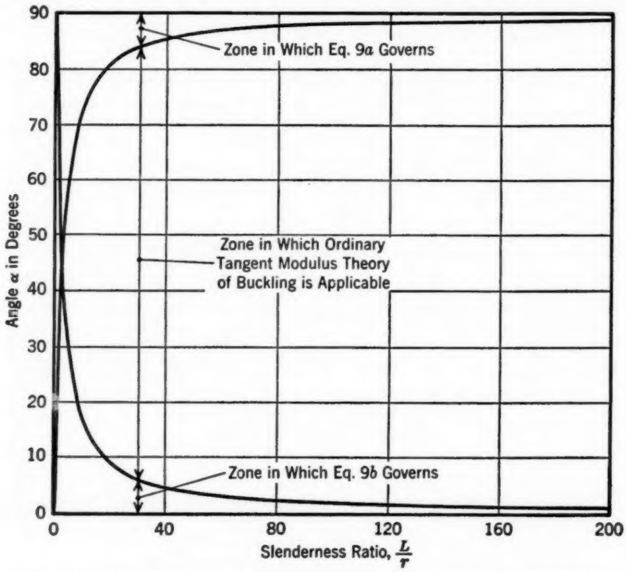

Fig. 14.—Applicability of Theories of Structural Instability

theory of plasticity. It merely emphasizes the fact that upper and lower limits for the solution of some apparently complicated problems can be obtained very simply by using existing theory. For the theoretician as well as for the practical engineer, it is important to insist that lack of a general theory does not invalidate a particular fact.

S. K. Ghaswala,[17] Assoc. M. ASCE.—An interesting elementary account of the theory of plasticity is presented and should be studied by all concerned with the subject.

The design of engineering structures is based on the solid foundation of the theory of elasticity, whose empirical derivation dates from the seventeenth century. The classic observations of Galileo in 1638 on the fracture of loaded beams[18] and Robert Hooke's experiments some years later on the resilience of springs[19] suggested physical hypotheses capable of mathematical formulation.

[17] Chartered Engr., Fort, Bombay, India.

[18] "Dialogues Concerning Two New Sciences" (English Translation), by Galileo Galilei, Northwestern Univ., Evanston and Chicago, Ill., 1939.

[19] "Lectures De Potentia Restitutiva," by Robert Hooke, published in "Early Science in Oxford," by R. T. Gunther, Vol. 8 (printed for author), Oxford, 1931.

As the years passed, the theory of elasticity continued to develop into an exact science and became part of mechanics and applied mathematics through the work of such pioneers as M. Navier, S. D. Poisson, A. L. Cauchy, M. B. de Saint-Venant, James and Daniel Bernoulli, Lord Kelvin, K. Pearson, P. G. Tait, and T. Young.[20,21] Today, for most of the simple design problems met by engineers, the scope and reliability of this theory has been amply demonstrated by the availability of exact results. During the past few years, however, intensive study has been directed towards evaluating the exact strength of composite and sandwich materials[22] and towards gaining a wider knowledge of elastically imperfect materials, of mechanical properties of ductile materials, of the crystal structure of solids, and of the behavior of metals during various fabricating processes. These efforts have led to a more satisfactory understanding of the importance of the plastic nature of solids and have necessitated the formulation of concepts not included in the theory of elasticity as propounded by its pioneers.

The behavior of material under stress so great that plastic action is set up is of interest in many spheres. Thus, in the bending, punching, and stretching of cold material, stresses beyond the yield point of the material develop; in the phenomenon of accidental overload in car couplers, draft rigging, and side frames of railroad rolling stock, plasticity becomes the governing factor in design. In problems concerning the stability of slender columns, stress concentration, and energy absorption, the theory of elasticity is no longer valid, and the theory of plasticity comes into play. Among problems of the latter class are structures built for air raid protection[23] and the elements of a torpedo protection system in which the ship's hull functions under explosive loading.[24]

In describing the theories of plasticity the author should have introduced the two concepts of plastic flow and plastic deformation more clearly so as to enable an uninitiated structural engineer to grasp the subject. The theory of plastic flow is based on the relation between stress and the rate of plastic strain, while the theory of plastic deformation is based on the relation between stress and plastic strain. The plastic flow theory is derived from the work of Saint-Venant, M. Levy, R. von Mises, L. Prandtl, T. Y. Lin, Assoc. M. ASCE, E. Reuss, and W. Prager, M. ASCE, and the plastic deformation theory is based on the work by A. Nádai, H. Hencky, and A. A. Ilyushin. The mathematical aspect of the subject has been ably treated by R. Hill.[25] In fact, from the standpoint of mathematics, the theory of plasticity forms one of the most active fields of research in the nonlinear mechanics of deformable media. The earliest statements of the concept of plastic design were made by N. C. Kist[26] and M. Gruning[27]; applications of the method to the design of transmission towers by

[20] "A History of the Elasticity and of Strength of Materials," by Isaac Todhunter and Karl Pearson, Cambridge Univ. Press, Cambridge, England, Vol. I, 1886.

[21] "A Treatise on the Mathematical Theory of Elasticity," by A. E. H. Love, Cambridge Univ. Press, Cambridge, England, 4th Ed., 1927.

[22] "Elements of Sandwich Construction," by S. K. Ghaswala, *Journal*, Inst. of Engrs., India, Vol. 31, No. 1, Sept., 1950, pp. 47–62.

[23] "The Civil Engineer in War," by J. F. Baker, Inst. of Civ. Engrs., London, England, Vol. 3, 1948.

[24] "Plastic Deformation, Principles and Theories," by L. N. Kachanov, N. M. Beliaev, A. A. Ilyushin, W. Mostow, and A. N. Gleyzal, Mapleton House, New York, N. Y., 1948.

[25] "The Mathematical Theory of Plasticity," by R. Hill, Oxford Univ. Press, London, England, 1950.

[26] "Die Zähigkeit des Materials als Grundlage für die Berechnung von Brücken, Hochbauten und ähnlichen Konstruktionen aus Flusseisen," by N. C. Kist, *Der Eisenbau*, Leipzig, Vol. 11, 1920, pp. 425–428.

[27] "Die Tragfähigkeit statisch unbestimmter Tragwerke aus Stahl bei beliebig häufig wiederholter Belastung," by M. Grüning, Julius Springer, Berlin, 1926.

C. M. Goodrich in 1910 are described by J. A. Van den Broek,[28] M. ASCE, and a general review of work in this field prior to 1931 was given by the late Friedrich Bleich,[29] M. ASCE. One of the most important projects undertaken in England in recent times in the application of the theory of plasticity to structural design problems is due to the efforts of J. F. Baker, Assoc. M. ASCE, and his colleagues at Cambridge University, Cambridge, England.[30,31,32,33] It is regretted that the author has not made any specific or detailed mention of this work as it would have added much to the value of the paper. The so-called plastic method of design evolved by Mr. Baker is of particular application to welded structures, such as portal frames, where its use in lieu of the elastic design method leads to a more economical design, simplification of design calculations, and, most important, places the actual method of calculation on a more rational basis by maintaining a uniform safety factor in all members. In addition to English and American research, a flourishing school of research in this field has been established in Russia, mainly through the work of Mr. Ilyushin, L. N. Kachanov, N. M. Beliaev, and others.[24]

The subject of plasticity is closely linked with "anelasticity," a word coined by Clarence Zener of the newly established Institute for the Study of Metals at the University of Chicago (Chicago, Ill.).[34] Anelasticity denotes that property of solids by which stress and strain are not single-valued functions of one another in that low stress range in which no permanent set occurs and in which the relation of stress to strain is still linear. All manifestations of anelasticity, such as creep, hysteresis, internal friction, stress relaxation, and variation of modulus with frequency of measurements are capable of being interpreted in a better manner by this specialized study. The writer would like to see some discussion of this topic, which he believes will exert a profound influence in the future on the thought and practice of structural engineering.[35]

D. C. DRUCKER,[36] Assoc. M. ASCE.—It has been made painfully clear by the discussion that the paper attempted to cover too broad a phase of plasticity in too brief a manner. The objective was to indicate some of the directions of special interest to the author in which the field has been developing. There was no intention of criticizing accepted design procedures in the vast majority of cases that have been checked repeatedly by practical experience and laboratory tests. Simple illustrations were employed to introduce a few fundamental con-

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[28] "Theory of Limit Design," by J. A. Van den Broek, John Wiley & Sons, Inc., New York, N. Y., 1948, p. 64.

[29] "Stahlhochbauten, ihre Theorie, Berechnung und Bauliche Gestaltung," by Friedrich Bleich, Julius Springer, Berlin, 1932.

[30] "The Calculation of Collapse Loads for Framed Structures," by B. G. Neal and P. S. Symonds, *Journal of the Institution of Civil Engineers*, London, England, No. 1 (1950–51), November, 1950, pp. 21–40.

[31] "The Design of Steel Frames," by J. F. Baker, *The Structural Engineer*, London, England, Vol. 27, October, 1949, pp. 397–431.

[32] "A Review of Recent Investigations into the Behaviour of Steel Frames in the Plastic Range," by J. F. Baker, *Journal of the Institution of Civil Engineers*, London, England, No. 3 (1948–49), January, 1949, pp. 188–240.

[33] "Theory of Plasticity—Elements of Simple Theory," by J. W. Roderick, *Philosophical Magazine and Journal of Science*, London, England, 7th Series, Vol. 39, July, 1948, pp. 529–539.

[34] "Elasticity and Anelasticity of Metals," by Clarence Zener, Univ. of Chicago Press, Chicago, Ill., 1948.

[35] "Atomic Structure of Engineering Metals," by S. K. Ghaswala, *Science and Culture*, Calcutta, India, February, 1947, pp. 388–395.

[36] Prof. of Eng., Brown Univ., Providence, R. I.

cepts to those unfamiliar with the theory. Clearly, the well-informed discussers are not in this category. The necessity for more advanced theory may not become apparent to the experienced engineer untill more elaborate problems are investigated.

However, a number of statements are made in the discussion indicating that in a few cases the inadequacy of past techniques and sometimes "engineering intuition" is not fully appreciated. Mr. Ghaswala implies that the design of engineering structures should be based on the "solid foundation of the theory of elasticity," except for structures which are intended to function in the plastic range. Elastic theory is certainly of great value but, as stated in the paper, design factors of safety have meaning only when consideration is given to what happens under overload—generally when plastic action takes place. In fact, the elastic theory, instead of being invalid, is of greatest value in problems of stress concentration when fatigue may occur and in problems of the buckling of slender members.

Mr. Ghaswala is critical of the definitions of deformation and of incremental (or flow) theory. To a considerable extent definitions are arbitrary, of course, but Mr. Ghaswala's definitions are far too restrictive. The facts are that deformation theory, which is basically incorrect, postulates (as stated in the paper under the heading "Mathematical Theories of Plasticity") "* * * that stress and strain are uniquely related as long as loading continues.* * *" Incremental theory, which is self-consistent, gives the "* * * increment in strain in terms of the existing state (stress, strain) and (strain) history and the changes in stress." These theories are described in considerable detail elsewhere.[9,10,12]

The author shares Mr. Ghaswala's high regard for the work of Mr. Baker and his colleagues at Cambridge University. He enjoyed a pleasant association with Bernard Neal and with Jacques Heyman, each of whom spent a year at Brown University, Providence, R. I., and returned to Cambridge. Close liaison between the Brown University group under Mr. Prager and the group under Mr. Baker has been materially aided by the two years spent by Paul Symonds in Cambridge.

There is complete agreement with Mr. Karol's opening remarks that a naive approach in particular cases is as good as a more formal one. The difficulty lies in deciding which cases are simple and which are not. It was one of the intentions of the paper to show possible stumbling blocks and caution signs. As an example, shear stress in the web of an I-beam is ignored in many limit design analyses today, and rightly so. However, Fig. 13 shows more clearly than the words of the paper that proper consideration of shear is important. The general question of placing upper and lower limits on the answer in more difficult examples is described in later papers.[37,38,39]

[9] "Recent Developments in the Mathematical Theory of Plasticity," by William Prager, *Journal of Applied Physics*, March, 1949, pp. 235–341.

[10] "The Significance of the Criterion for Additional Plastic Deformation of Metals," by D. C. Drucker, *Journal of Colloid Science* (rheology issue), Vol. 4, 1949, pp. 299–311.

[12] "The Relation of Experiments to Mathematical Theories of Plasticity," by D. C. Drucker, *Journal of Applied Mechanics*, Vol. 16, 1949, pp. 349–357.

[37] "Limit Design of Beams and Frames," by H. J. Greenberg and W. Prager, *Proceedings-Separate No. 59*, ASCE, Vol. 77, February, 1951.

[38] "The Safety Factor of an Elastic-Plastic Body in Plane Strain," by D. C. Drucker, H. J. Greenberg, and W. Prager, to appear in the *Journal of Applied Mechanics* (publication pending).

[39] "Extended Limit Design Theorems for Structures and Continuous Media," by D. C. Drucker, H. J. Greenberg, and W. Prager, to appear in the *Quarterly of Applied Mathematics* (publication pending).

Mr. Karol has made a very able analysis of the buckling of the simple frame in Fig. 7. Fig. 14 demonstrates admirably that individual buckling of the bars usually governs. But, like Eq. 9*b*, it also shows that no matter how small the l/r ratio is made, buckling is still important. This is another point that might be overlooked. Here again the illustration in the paper was to acquaint civil engineers with this problem, not to derive design rules for particular structures. Also, even though Shanley's concept most often leads to loads that are close to the reduced modulus loads, it does not always do so. Furthermore, his physical idea is the proper one and will certainly be employed in the future.

Mr. Steed is entirely correct in his statement that simple limit design of pin-connected trusses is not permissible. Buckling must be taken into account; in the case of structural steel it is known that after plastic buckling the load drops as the member is shortened. The history of the stress in each member must be considered in some way, or in some cases it may be possible to prestress the truss to avoid premature buckling. It would appear, however, that the technique described by Mr. Steed becomes very involved for a highly indeterminate structure if a design for minimum weight is desired. If a combination of Mr. Steed's method and an upper and lower bound procedure could be found, it would appear to have real promise for truly complicated framed structures.

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